

# Spatial Coherence and Crest-Length Statistics of Waves in Deep Water

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## LONG-TERM GOALS

The long-term objective is to determine how nonlinear interactions and directional spreading affect the spatial coherence and crest-length statistics of ocean surface gravity waves.

## OBJECTIVES

To investigate the spatial coherence and statistical distribution of crest lengths of deep water ocean waves we have been developing and verifying with field observations the methodology to simulate two-dimensional sea surfaces with a specified frequency-directional spectrum and local nonlinearity described by theory. Our specific objectives are to

- simulate realizations of the two-dimensional sea surface given a specified frequency-directional spectrum and incorporating local nonlinearity based on second-order theory
- estimate crest-length statistics from the simulated sea surfaces for a range of wave conditions
- compare simulated with remotely-sensed sea surfaces

## APPROACH

We have developed the methodology to simulate realizations of a nonlinear sea surface with a specified ('target') frequency-directional or wavenumber spectrum and a specified bispectrum. The bispectrum describes statistically the phase relationships between triads of nonlinearly interacting waves, for example the quadratic difference interaction that couples two swell waves (with wavenumbers  $k_p$  and  $k_p + \delta$  where  $\delta$  is a small number) and a low frequency long wave with wavenumber  $\delta$ ), or sum interactions that couple swell with higher harmonics. The simulation algorithm accounts for interactions between waves traveling in different directions. The target wavenumber spectrum can be obtained from observations, the output of a wave prediction model, or theory. The target bispectrum can be obtained from observations or from second-order nonlinear theory given the wavenumber spectrum.

To test second-order nonlinear theory, statistics of the observed wave field (eg, higher-order spectra, sea-surface elevation skewness, crest length) are compared with statistics from sea surfaces simulated from the observed wavenumber spectrum and the corresponding bispectrum consistent with second-order theory.

## **WORK COMPLETED**

Software to estimate higher-order spectra of sea surfaces has been produced. In addition, the theory to determine the bispectrum from the wavenumber spectrum estimated with observations from a moving platform (eg, an airplane) has been developed. Simulations with narrow (Figure 1) and broad (Figure 2) frequency-directional spectra show differences in the corresponding sea surfaces. In particular, the groups are larger (more big waves in a row) and the crests are longer for narrow band sea surfaces than for the broad band waves.

Wavenumber and higher-order spectra have been estimated from LIDAR observations (provided by P. Hwang, NRL) of the sea surface. Theoretical bispectra have been calculated from the observed wavenumber spectra, and both theoretical and observed bispectra have been used as input to simulate realizations of the sea surface.

## **RESULTS**

The bispectrum estimated from the observed wavenumber spectrum and second-order nonlinear theory compares well with the bispectrum estimated directly from LIDAR images of approximately 1 m high waves in 20-30 m water depth near the North Carolina coast. Third-order statistics of the sea surfaces obtained from numerical simulations using the observed wavenumber spectrum and either the observed or theoretical bispectrum compare well with the statistics of the observed sea surface, verifying both the simulation technique and second-order nonlinear theory.

## **IMPACT/APPLICATIONS**

An impact of this research is the verification of second-order theory for directionally spread waves, allowing realistic nonlinear sea surfaces to be simulated given the wavenumber spectrum.

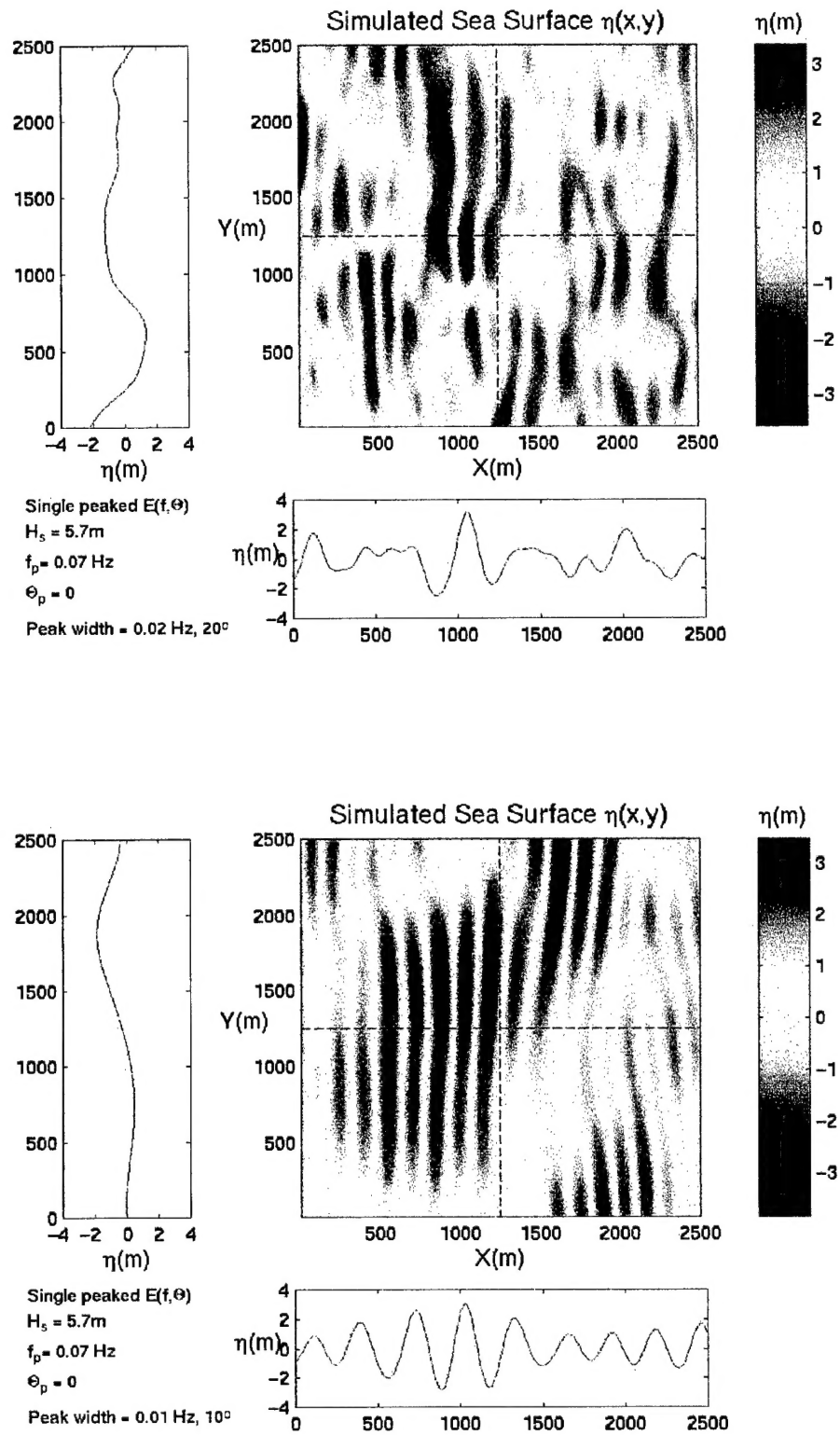
## **TRANSITIONS**

## **RELATED PROJECTS**

The research performed here is related to our investigations of the evolution of waves across the continental shelf (SHOWEX) and the nearshore and surfzone (SandyDuck).

## **PUBLICATIONS**

Herbers, T.H.C, Steve Elgar, N.A. Sarap, and R.T.Guza, 2000 Dispersion properties of surface gravity waves in shallow water, *J. Physical Oceanography*, submitted.



**Figure 1.** Sea surfaces consisting of lowest-order wind generated waves with broad (upper) and narrow (lower) frequency-directional spectra, and second-order nonlinear waves consistent with theory. The amount of nonlinear coupling for each pair of wind waves and the corresponding sum or difference second-order wave is a function of the wavenumber spectrum and the frequencies of the first-order wind waves. The significant wave height is 5.7 m. The colors are sea-surface elevation (red is wave crests, dark blue is troughs). The small panels on the left and bottom of the color contours are 1D cuts through the center of the sea surface (dotted lines on the sea surfaces show the locations of the 1D surfaces).

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